Extra Example 6

Find the number of distinguishable permutations of the letters in (a) SOCCER and (b) SWIMMING.
360; 10,080

Key Questions to Ask for Example 6

- Why does it make sense to divide by the permutations of repeated letters? **If a letter repeats t times, then $t!$ of the permutations will be identical.**
- Why do the fractions not include factors of 1! for each nonrepeated letter of the word? **Since $1! = 1$, dividing by 1! does not affect the value.**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you determine the number of distinguishable permutations in the letters of a word?

- **If one event can occur in m ways and another event can occur in n ways, then the number of ways that both events can occur is $m \cdot n$.**
- **The number of permutations of n distinct objects is $n!$.**
- **You can use the formula $_nP_r = \frac{n!}{(n-r)!}$ to determine the number of permutations of r objects taken from a group of n distinct objects.**

The number of distinguishable permutations of n objects where one object is repeated s_1 times, another is repeated s_2 times, and so on for k items, is $\frac{n!}{s_1! \cdot s_2! \cdot \ldots \cdot s_k!}$.

3–6. See Additional Answers beginning on p. AA1.

EXAMPLE 6 Find permutations with repetition

Find the number of distinguishable permutations of the letters in (a) MIAMI and (b) TALLAHASSEE.

Solution

a. MIAMI has 5 letters of which M and I are each repeated 2 times. So, the number of distinguishable permutations is $\frac{5!}{2! \cdot 2!} = \frac{120}{2 \cdot 2} = 30$.

b. TALLAHASSEE has 11 letters of which A is repeated 3 times, and L, S, and E are each repeated 2 times. So, the number of distinguishable permutations is $\frac{11!}{3! \cdot 2! \cdot 2! \cdot 2!} = \frac{39{,}916{,}800}{6 \cdot 2 \cdot 2 \cdot 2} = 831{,}600$.

✓ GUIDED PRACTICE for Example 6

Find the number of distinguishable permutations of the letters in the word.

8. MALL **12**

9. KAYAK **30**

10. CINCINNATI **50,400**

10.1 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 13, 35, and 65

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 42, 55, 57, and 68

SKILL PRACTICE

A

1. VOCABULARY What is a permutation of n objects? **The number of ways n objects can be ordered.**

2. ★ WRITING Simplify the formula for $_nP_r$ when $r = 0$. *Explain* why this result makes sense. ***Sample answer:*** **When $r = 0$, the permutation simplifies to $\frac{n!}{n!}$, which is equal to 1.**

EXAMPLE 1 on p. 682 for Exs. 3–6

TREE DIAGRAMS **An object has an attribute from each list. Make a tree diagram that shows the number of different objects that can be created.** **3–6. See margin.**

3.

T-Shirts
Size: M, L, XL
Type: long-sleeved, short-sleeved

4.

Toast
Bread: white, wheat
Spread: jam, margarine

5.

Meal
Entrée: chicken, fish, pasta
Side: corn, green beans, potato

6.

Furniture
Wood: cherry, mahogany, oak, pine
Finish: stained, painted, unfinished

EXAMPLE 2 on p. 683 for Exs. 7–10

FUNDAMENTAL COUNTING PRINCIPLE **Each event can occur in the given number of ways. Find the number of ways all of the events can occur.**

7. Event A: 2 ways; Event B: 4 ways **8 ways**

8. Event A: 5 ways; Event B: 2 ways **10 ways**

9. Event A: 4 ways; Event B: 3 ways; Event C: 5 ways **60 ways**

10. Event A: 3 ways; Event B: 6 ways; Event C: 5 ways; Event D: 2 ways **180 ways**

EXAMPLE 3 on p. 683 for Exs. 11–17

LICENSE PLATES **For the given configuration, determine how many different license plates are possible if (a) digits and letters can be repeated, and (b) digits and letters cannot be repeated.** 11–16. See margin.

11. 4 letters followed by 3 digits

12. 2 letters followed by 5 digits

13. 4 letters followed by 2 digits

14. 5 digits followed by 3 letters

15. 1 digit followed by 5 letters

16. 6 letters

17. ★ **MULTIPLE CHOICE** How many different license plates with 2 letters followed by 4 digits are possible if digits and letters cannot be repeated? A

Ⓐ 3,276,000 Ⓑ 6,760,000 Ⓒ 32,292,000 Ⓓ 45,697,600

EXAMPLES 4 and 5 on pp. 684–685 for Exs. 18–41

FACTORIALS **Evaluate the expression.**

18. $7!$ 5040

19. $11!$ 39,916,800

20. $1!$ 1

21. $8!$ 40,320

22. $4!$ 24

23. $0!$ 1

24. $12!$ 479,001,600

25. $6!$ 720

26. $3! \cdot 4!$ 144

27. $3(4!)$ 72

28. $\frac{8!}{(8-5)!}$ 6720

29. $\frac{9!}{4! \cdot 4!}$ 630

PERMUTATIONS **Find the number of permutations.**

30. ${}_4P_4$ 24

31. ${}_6P_2$ 30

32. ${}_{10}P_1$ 10

33. ${}_8P_7$ 40,320

34. ${}_7P_4$ 840

35. ${}_9P_2$ 72

36. ${}_{13}P_8$ 51,891,840

37. ${}_7P_7$ 5040

38. ${}_5P_0$ 1

39. ${}_9P_4$ 3024

40. ${}_{11}P_4$ 7920

41. ${}_{15}P_0$ 1

42. ★ **SHORT RESPONSE** Let n be a positive integer. Find the number of permutations of n objects taken $n - 1$ at a time. *Compare* your answer with the number of permutations of all n objects. Does this make sense? *Explain.*
$n!$; they are the same; yes; the permutation simplifies to $\frac{n!}{1!}$, which is equal to $n!$.

EXAMPLE 6 Ⓑ on p. 686 for Exs. 43–55

PERMUTATIONS WITH REPETITION **Find the number of distinguishable permutations of the letters in the word.**

43. OFF 3

44. TREE 12

45. SKILL 60

46. YELLOW 360

47. GRAVEL 720

48. PANAMA 120

49. ARKANSAS 3360

50. FACTORIAL 181,440

51. MAGNETIC 40,320

52. HONOLULU 5040

53. CLEVELAND 90,720

54. MISSISSIPPI 34,650

55. ★ **MULTIPLE CHOICE** What is the number of distinguishable permutations of the letters in the word HAWAII? B

Ⓐ 24 Ⓑ 180 Ⓒ 360 Ⓓ 720

56. **ERROR ANALYSIS** In bingo, balls labeled from 1 to 75 are drawn from a container without being replaced. *Describe* and correct the error in finding the number of ways the first 4 numbers can be chosen for a game of bingo.

$75 \cdot 75 \cdot 75 \cdot 75$
$= 31{,}640{,}625$ ✗

56. The numbers are not replaced, so the number of balls must decrease by 1 after each drawing; $75 \cdot 74 \cdot 73 \cdot 72 = 29{,}170{,}800$. Ⓒ

57. ★ **SHORT RESPONSE** *Explain* how the fundamental counting principle can be used to justify the formula for the number of permutations of n distinct objects. See margin.

SOLVING EQUATIONS **Solve for n.**

58. ${}_nP_4 = 8({}_nP_3)$ 11

59. ${}_nP_6 = 5({}_nP_5)$ 10

60. ${}_nP_5 = 9({}_nP_4)$ 13

61. **CHALLENGE** Find the number of distinguishable permutations of 6 letters that are chosen from the letters in the word MANATEE. 1260

4 PRACTICE AND APPLY

Assignment Guide

Answer Transparencies available for all exercises

Basic:
Day 1: pp. 686–689
Exs. 1–4, 7, 8, 11–13, 17–21, 30–33, 42–46, 62–68, 72, 75, 78, 81, 84, 87

Average:
Day 1: pp. 686–689
Exs. 1, 2, 4, 5, 8, 9, 13, 14, 17, 22–25, 34–37, 42, 47–50, 55, 56, 62–70, 73, 77, 79, 82, 85, 88

Advanced:
Day 1: pp. 686–689
Exs. 1, 2, 6, 10, 15–17, 26–29, 38–42, 51–61*, 63, 65–71*, 74, 80, 86, 89

Block:
pp. 686–689
Exs. 1, 2, 4, 5, 8, 9, 13, 14, 17, 22–25, 34–37, 42, 47–50, 55, 56, 62–70, 73, 77, 79, 82, 85, 88 (with 10.2)

Differentiated Instruction

See *Algebra 2 Best Practices Toolkit* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 4, 12, 20, 32, 64

Average: 14, 24, 36, 48, 65

Advanced: 16, 28, 40, 52, 66

Extra Practice

- Student Edition, p. 1019
- Chapter 10 Resource Book: Practice levels A, B, C, pp. 6–8

Practice Worksheet

An easily-readable reduced practice page (with answers) for this lesson can be found on p. 680C.

11a. 456,976,000 license plates
11b. 258,336,000 license plates
12a. 67,600,000 license plates
12b. 19,656,000 license plates
13a. 45,697,600 license plates
13b. 32,292,000 license plates
14a. 1,757,600,000 license plates
14b. 471,744,000 license plates
15a. 118,813,760 license plates
15b. 78,936,000 license plates
16a. 308,915,776 license plates
16b. 165,765,600 license plates
57. See Additional Answers beginning on p. AA1.